Repeated Fair Allocation of Indivisible Items

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Abstract

In practice, items are not always allocated once and for all, but often repeatedly. For example, when the items are recurring chores to distribute in a household. Motivated by this, we initiate the study of the repeated fair division of indivisible items.

Applications

- Fairly distributing household chores between a couple
- Allocating teaching duties to professors over the semesters
- Granting employees daily access to a common infrastructure

Repetition: Why Bother?

In the one-shot setting, a **Proportional** (let alone Envy-Free) and **Pareto-Optimal** allocation may not exist. Our main goal:

"Can we guarantee better fairness and efficiency properties by looking at the repeated allocation of items?"

Main Idea

Suppose that we want to allocate a single item ▲ between two agents, 👤 (red) and 👤 (blue). Problem:

👤 (red)	👤 (blue)
▲	

not fair!

👤 (red)	👤 (blue)

not efficient!

What if we share them over time?

	👤 (red)	👤 (blue)
day 1	▲	
day 2		▲

Each day's allocation is not fair, but the overall allocation is!

Formal Model

We have n agents (👤, 👤, 👤, ...) that need to share some items (▲, ■, ★, ...). Agents have additive utilities:

	👤 (red)	👤 (blue)	👤 (orange)
▲	1	3	4
■	5	2	1
★	-3	-4	-2

We have k time-steps at our disposal. Example ($k = 3$):

	👤 (red)	👤 (blue)	👤 (orange)
day 1	■★	▲	
day 2	■		▲★
day 3	■		▲★

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Axioms

An axiom can be satisfied overall (while looking globally at the whole bundle, over all time-steps) or per round (if it is satisfied individually by all time-steps).

- **Envy-freeness** (EF): No agent prefers someone else's bundle
- **Envy-freeness up to one item** (EF1): If an agent envies some other agent, we can eliminate envy by removing one item from the bundle of one of the two agents
- **Proportionality** (PR): Each agent receives at least $1/n$ of their evaluation of the whole set of items
- **Pareto-optimality** (PO): There is no reallocation that is strictly better for some agents, and worse for none

Results: General Case

Under certain conditions, envy-freeness is always achievable:

> *If k is a multiple of n, an **overall EF** allocation always exists.*

To achieve this, we can rotate the items at each time-step, e.g.:

	👤 (red)	👤 (blue)	👤 (orange)
day 1	▲	■	★
day 2	★	▲	■
day 3	■	★	▲

What about efficiency? Even if k is a multiple of n, an overall EF and PO allocation might not exist. Still:

> *If k is a multiple of n, an **overall PR and PO** allocation always exists.*

Results: Two-agent Case

For two agents, we have stronger fairness guarantees:

> *For two agents, if k is even, an **overall EF and PO** allocation always exists.*

What about the individual time-steps? We cannot have envy-freeness in every round. However:

> *For two agents, if k is even, an allocation which is **overall EF** and **EF1 per round** always exists.*

Can we additionally have efficiency? Not if $k > 2$, but:

> *For two agents, if $k = 2$, we can always find an **overall EF and PO** allocation that is **EF1 per round**.*

> *For two agents, if k is even, we can always find an **overall EF and PO** allocation that is **weakly EF1 per round**.*

Results: Variable Number of Rounds

What if the number of rounds is not known in advance? Via a connection to the randomised and divisible settings, we show:

> *For every utility profile, there is some k for which an **overall EF and PO** allocation that is **PROP[1, 1] per round** exists.*